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The Weber Force

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Abstract: In this paper It will be shown how the Weber Force law emerges as an averaged effect between two electrically neutral closed current loops under the assumption that the force between current elements is given by the Lorentz Force operating on the Four Potential as prescribed by the full retarded form of Maxwell's Equations. That is it will be shown that the total force given by the Weber Force between two neutral closed current loops is experimentally indistinguishable from the total force computed by The Lorentz interaction. As a corollary it will be shown how only under the conditions of non neutrality and open circuits, how one force principle might diverge from the other in their respective predictions of the interaction characteristics in such a configuration. Under such conditions this therefore presents the possibility, through experimental means, of how to determine which force law is actually at work in nature.

Introduction

The three primary physical phenomena which provide the basis for the formulation of Maxwell's Electromagnetism are firstly the Coulomb Force which acts reciprocally between static electric charges, the mechanical force which is induced on a current element in the presence of a magnetic field, itself the product of separate current sources, and thirdly the induced force, or EMF, as expressed in Faraday's law of Induction. All three forces are summarized in what is known as the Lorentz force law formed from the derivatives of the Four Potential and the local velocity field of a charge distribution on which the Lorentz Force acts, and with of course the Four Potential obeying the wave equation. However prior to Maxwell's field based formulation, their already existed a unifying principle which already described the above mentioned phenomena in all details and which first made its appearance back in 1847 when Wilhelm Weber along with his collaborator, Carl Fredrick Gauss, and on the basis of Ampere's law, formulated his ballistic force law which was equal in power to that of Maxwell's theory in its ability to describe all known electromagnetic phenomena. For various reasons, primarily pertaining to considerations of a conceptual nature, Weber's theory vanished from the popular scientific paradigm in favour of Maxwell's field approach. For an excellent modern treatment of Weber's theory see [5] and also for an application of The Weber Force to gravity see [6]. It should also be mentioned, in regard to describing gravitation, that other successful attempts employing Weber like force laws, were made including one by Schrödinger which like General Relativity could explain the motion of the planet Mercury [7]. In this exposition it will be shown how under certain conditions to be elucidated, that the formulations of both Maxwell and Weber are indistinguishable in terms of their description of the three primary phenomena as described above. As a corollary It will be then suggested how each formulation should differ in their respective predictions of certain phenomena when the above mentioned conditions that are to be elucidated, are no longer satisfied.

The Lorentz Force

Evaluation of $\boldsymbol{u}\times\boldsymbol{B}(\boldsymbol{x},\mathrm{t})$. According to the Lorentz Force, the total external force, $\boldsymbol{F}_{\mathrm{V}'}$**,** acting on a body of charge contained within the volume V', is given by;

$$\boldsymbol{F}_{\mathrm{V}'}=\int_{\mathrm{V}'}\rho_e(\boldsymbol{x},t)\big(\boldsymbol{E}(\boldsymbol{x},\mathrm{t})+\boldsymbol{u}\times\boldsymbol{B}(\boldsymbol{x},\mathrm{t})\big)\delta x_1\delta x_2\delta x_3. \qquad (1.1)$$

It is also worthwhile to state at this point that $\boldsymbol{F}_{\mathrm{V}'}$ is induced by the action of a second body of charge external to V' and which is contained within a volume V, so that $(\Phi,\mathbf{A})$ represents the four potential due to the charge distribution contained within V, such that;

$$\boldsymbol{E}(\boldsymbol{x},\mathrm{t})=-\nabla\Phi(\boldsymbol{x},\mathrm{t})-\frac{\partial\mathbf{A}(\boldsymbol{x},t)}{\partial t} \qquad \boldsymbol{B}(\boldsymbol{x},\mathrm{t})=\nabla_{\boldsymbol{x}}\times\mathbf{A}(\boldsymbol{x},t) \qquad (1.2)$$

$$\begin{aligned}A_{\mathrm{j}}(\boldsymbol{x},t)&=\frac{1}{4\pi\mathrm{c}^2}\int_{\mathrm{V}}\rho_e\left(\boldsymbol{y},t-\frac{r}{c}\right)\frac{u_j\left(\boldsymbol{y},t-\frac{r}{c}\right)}{r}\delta y_1\delta y_2\delta y_3\\&\approx\frac{1}{4\pi\mathrm{c}^2}\int_{\mathrm{V}}\rho_e(\boldsymbol{y},t)\frac{u_j(\boldsymbol{y},t)}{r}\delta y_1\delta y_2\delta y_3+\mathrm{O}\left(\frac{1}{c^3}\right) \qquad (1.3)\end{aligned}$$

$$\Phi(\boldsymbol{x},\mathrm{t})=\frac{1}{4\pi}\int_{\mathrm{V}}\rho_e\left(\boldsymbol{y},t-\frac{r}{c}\right)\frac{1}{r}\delta y_1\delta y_2\delta y_3. \qquad (1.4)$$

At this point it will be necessary to introduce some definitions and conventions in regard to total and partial derivatives of an arbitrary continuous function $f(\boldsymbol{x},\boldsymbol{y},t)$. Define;

$$\begin{aligned}\frac{Df(\boldsymbol{x},\boldsymbol{y},t)}{Dt}&=\frac{\partial f}{\partial t}+\frac{\partial f}{\partial y_i}\frac{dy_i}{dt}+\frac{\partial f}{\partial x_i}\frac{dx_i}{dt}=\frac{\partial f}{\partial t}+\nabla_{\mathrm{y}}\mathrm{f}.\boldsymbol{u}(\boldsymbol{y},t)+\nabla_{\mathrm{x}}\mathrm{f}.\boldsymbol{u}(\boldsymbol{x},t)\\&=\frac{df(\boldsymbol{x},\boldsymbol{y},t)}{dt}+\nabla_{\mathrm{x}}\mathrm{f}.\boldsymbol{u}(\boldsymbol{x},t) \quad (1.5)\end{aligned}$$

$$\frac{df(\boldsymbol{x},\boldsymbol{y},t)}{dt}=\frac{\partial f}{\partial t}+\nabla_{\mathrm{y}}\mathrm{f}.\boldsymbol{u}(\boldsymbol{y},t) \qquad (1.6)$$

Armed with these definitions we can proceed as follows.

It is useful to observe that;

$$\frac{u_j(\boldsymbol{y},t)}{r}=\frac{\frac{dy_i}{dt}}{r}=\frac{\frac{d(y_j-x_j)}{dt}}{r}=\frac{\frac{dz_j}{dt}}{r}\ \ since\ \ \frac{d\boldsymbol{x}}{dt}=0 \qquad (1.7)$$

in which

$$z_j = y_j - x_j. \qquad (1.8)$$

Observing that

$$\frac{\frac{dz_j}{dt}}{r} = \frac{d(\frac{z_j}{r})}{dt} + \frac{z_j}{r^2}\frac{dr}{dt} \qquad (1.9)$$

allows for the possibility to express $\mathbf{A}(\boldsymbol{x},t)$ in a form which will be more relevant to the context of the analysis to follow. Combining Eq (1.9) and Eq (1.3) yields;

$$\mathbf{A}(\boldsymbol{x},t) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{d\hat{\boldsymbol{r}}}{dt} + \hat{\boldsymbol{r}}\frac{\frac{dr}{dt}}{r})\delta y_1\delta y_2\delta y_3 \qquad (1.10)$$

In which the unit vector $\hat{\boldsymbol{r}}$ is given by $\hat{\boldsymbol{r}} = (\frac{z_1}{r}, \frac{z_2}{r}, \frac{z_3}{r})$ and the magnitude $r = |\boldsymbol{x} - \boldsymbol{y}|$, is simply the Euclidean distance between the points $\boldsymbol{x}$ and $\boldsymbol{y}$.

From the definition of $\boldsymbol{B}(\boldsymbol{x},\mathrm{t})$ we can write;

$$\boldsymbol{B}(\boldsymbol{x},\mathrm{t}) = \nabla_{\boldsymbol{x}} \times \mathbf{A}(\boldsymbol{x},t) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\nabla_{\boldsymbol{x}} \times \left(\frac{d\hat{\boldsymbol{r}}}{dt} + \hat{\boldsymbol{r}}\frac{\frac{dr}{dt}}{r}\right))\delta y_1\delta y_2\delta y_3. \qquad (1.11)$$

Utilizing Eq (B.2) and Eq (B.3) then yields;

$$\boldsymbol{B}(\boldsymbol{x},\mathrm{t}) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\hat{\boldsymbol{r}} \times \frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt})\delta y_1\delta y_2\delta y_3. \qquad (1.12)$$

Finally by employing Eq (B.12), the Electrodynamic intensity , $\boldsymbol{u} \times \boldsymbol{B}(\boldsymbol{x},\mathrm{t})$, can be expressed as;

$$\begin{aligned}\boldsymbol{u} \times \boldsymbol{B}(\boldsymbol{x},\mathrm{t}) &= \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \times \left(\hat{\boldsymbol{r}} \times \frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt}\right))\delta y_1\delta y_2\delta y_3 \\ &= \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\left(\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt}\right)\hat{\boldsymbol{r}} - (\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}})\frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt})\delta y_1\delta y_2\delta y_3. \qquad (1.13)\end{aligned}$$

Now since

$$\begin{aligned}\frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right) &= (\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}})\frac{d}{dt}\left(\frac{\hat{\boldsymbol{r}}}{r}\right) + \frac{\hat{\boldsymbol{r}}}{r}(\boldsymbol{u} \cdot \hat{\boldsymbol{r}}) \\ &= (\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}})\left(\frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt} - \frac{\hat{\boldsymbol{r}}}{r^2}\frac{dr}{dt}\right) + \frac{\hat{\boldsymbol{r}}}{r}\frac{d}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}}) \qquad (1.14)\end{aligned}$$

which is the same as;

$$-(\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}})\frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt} = -(\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r^2}\frac{dr}{dt} + \frac{\hat{\boldsymbol{r}}}{r}\frac{d}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}}) - \frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \cdot \hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right) \qquad (1.15)$$

means that we can rewrite Eq (1.13) as

$$\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\times\boldsymbol{B}(\boldsymbol{x},\mathrm{t})$$
$$=\frac{1}{4\pi c^2}\int_V\rho_e(\boldsymbol{y},t)(\left(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt}\right)\hat{\boldsymbol{r}}-(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r^2}\frac{dr}{dt}+\frac{\hat{\boldsymbol{r}}}{r}\frac{d}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})$$
$$-\frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right))\delta y_1\delta y_2\delta y_3. \qquad (1.16)$$

At this point, and primarily for the purposes of generating a thrust towards the emergence of the Weber Force, it will be necessary to rewrite some of the terms in Eq (1.16) in terms of the total derivatives, $\frac{Dr}{Dt}$ and $\frac{D^2r}{Dt^2}$, such that;

$$\frac{Dr}{Dt}=\frac{dr}{dt}-\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}} \qquad (1.17)$$

$$(\frac{Dr}{Dt})^2=(\frac{dr}{dt})^2-2\frac{dr}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})+(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})^{\mathbf{2}} \qquad (1.18)$$

$$\frac{D^2r}{Dt^2}=\frac{d}{dt}\left(\frac{dr}{dt}-\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}}\right)+\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\nabla_{\boldsymbol{x}}\left(\frac{dr}{dt}-\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}}\right)$$
$$=\frac{d^2r}{dt^2}-\frac{d}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})-\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\frac{d\hat{\boldsymbol{r}}}{dt}-\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\nabla_{\boldsymbol{x}}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}}). \qquad (1.19)$$

Inserting Eq (1.17) – Eq (1.19) into the following yields;

$$\left(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt}\right)\hat{\boldsymbol{r}}-(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r^2}\frac{dr}{dt}+\frac{\hat{\boldsymbol{r}}}{r}\frac{d}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})-\frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)$$
$$=\frac{\hat{\boldsymbol{r}}}{r}\left(\frac{d}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})+\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\frac{d\hat{\boldsymbol{r}}}{dt}-(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{1}{r}\frac{dr}{dt}\right)-\frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)$$
$$=\frac{\hat{\boldsymbol{r}}}{r^2}\left(r\frac{d^2r}{dt^2}-r\frac{D^2r}{Dt^2}-\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\nabla_{\boldsymbol{x}}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})r-(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{dr}{dt}\right)$$
$$-\frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)$$
$$=\frac{\hat{\boldsymbol{r}}}{r^2}\left(r\frac{d^2r}{dt^2}-r\frac{D^2r}{Dt^2}-\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\nabla_{\boldsymbol{x}}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})r+\frac{1}{2}(\frac{Dr}{Dt})^2-\frac{1}{2}(\frac{dr}{dt})^2\right.$$
$$\left.-\frac{1}{2}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})^{\mathbf{2}}\right)-\frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right). \qquad (1.20)$$

Inserting Eq (1.20) into Eq (1.16) then gives;

$$\boldsymbol{u}(\boldsymbol{x},\mathrm{t}) \times \boldsymbol{B}(\boldsymbol{x},\mathrm{t})$$

$$= \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\left(\frac{\hat{\boldsymbol{r}}}{r^2}\left(r\frac{d^2r}{dt^2} - r\frac{D^2r}{Dt^2} - \boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\nabla_{\boldsymbol{x}}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})r + \frac{1}{2}\left(\frac{Dr}{Dt}\right)^2\right.\right.$$

$$\left.\left. - \frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{1}{2}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})^2\right) - \frac{d}{dt}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)\right)\delta y_1\delta y_2\delta y_3$$

$$= \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\left(\frac{\hat{\boldsymbol{r}}}{r^2}\left(-c^2 + \frac{1}{2}\left(\frac{Dr}{Dt}\right)^2 - r\frac{D^2r}{Dt^2}\right)\right)\delta y_1\delta y_2\delta y_3$$

$$- \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{\hat{\boldsymbol{r}}}{r^2}\left(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\nabla_{\boldsymbol{x}}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})r + \frac{1}{2}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})^2\right.$$

$$\left. + \left(\frac{\partial\boldsymbol{u}(\boldsymbol{x},\mathrm{t})}{\partial t}\cdot\hat{\boldsymbol{r}}\right)r\right)\delta y_1\delta y_2\delta y_3$$

$$+ \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{\hat{\boldsymbol{r}}}{r^2}\left(c^2 + r\frac{d^2r}{dt^2} - \frac{1}{2}\left(\frac{dr}{dt}\right)^2\right)\delta y_1\delta y_2\delta y_3$$

$$- \frac{1}{4\pi c^2}\int \rho_e(\boldsymbol{y},t)\nabla_{\boldsymbol{y}}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)\cdot\boldsymbol{u}(\boldsymbol{y},\mathrm{t})\delta y_1\delta y_2\delta y_3$$

$$= \boldsymbol{W}(\boldsymbol{x},t,\boldsymbol{u}) - \boldsymbol{L}(\boldsymbol{x},t,\boldsymbol{u}) + \boldsymbol{E}'(\boldsymbol{x},t) - \boldsymbol{E}(\boldsymbol{x},\mathrm{t}) - \boldsymbol{C}(\boldsymbol{x},t,\boldsymbol{u}) \qquad (1.21)$$

in which

$$\boldsymbol{W}(\boldsymbol{x},t,\boldsymbol{u}) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{\hat{\boldsymbol{r}}}{r^2}\left(-c^2 + \frac{1}{2}\left(\frac{Dr}{Dt}\right)^2 - r\frac{D^2r}{Dt^2}\right))\delta y_1\delta y_2\delta y_3 \qquad (1.22)$$

$$\boldsymbol{L}(\boldsymbol{x},t,\boldsymbol{u}) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{\hat{\boldsymbol{r}}}{r^2}\left(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\nabla_{\boldsymbol{x}}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})r + \frac{1}{2}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})^2\right.$$

$$\left. + \left(\frac{\partial\boldsymbol{u}(\boldsymbol{x},\mathrm{t})}{\partial t}\cdot\hat{\boldsymbol{r}}\right)r\right)\delta y_1\delta y_2\delta y_3 \qquad (1.23)$$

$$\boldsymbol{E}'(\boldsymbol{x},t) - \boldsymbol{E}(\boldsymbol{x},\mathrm{t}) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{\hat{\boldsymbol{r}}}{r^2}\left(c^2 + r\frac{d^2r}{dt^2} - \frac{1}{2}\left(\frac{dr}{dt}\right)^2\right)\delta y_1\delta y_2\delta y_3 \qquad (1.24)$$

$$\boldsymbol{C}(\boldsymbol{x},t,\boldsymbol{u}) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\nabla_{\boldsymbol{y}}\left((\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)\cdot\boldsymbol{u}(\boldsymbol{y},\mathrm{t})\delta y_1\delta y_2\delta y_3. \qquad (1.25)$$

Finally it is now possible to write Eq (1.1) in a form which most facilitates the emergence of the sought after insight motivated by the preceding analysis. Using Eq (1.22) – Eq (1.25) allows Eq (1.1) to take the following form;

$$\boldsymbol{F}_{V'} = \int_{V'} \rho_e(\boldsymbol{x},t)\big(\boldsymbol{W}(\boldsymbol{x},t,\boldsymbol{u}) - \boldsymbol{L}(\boldsymbol{x},t,\boldsymbol{u}) + \boldsymbol{E}'(\boldsymbol{x},t) - \boldsymbol{C}(\boldsymbol{x},t,\boldsymbol{u})\big)\delta x_1\delta x_2\delta x_3. \qquad (1.26)$$

It will now be demonstrated, provided the following conditions are satisfied, that the action of the Lorentz and Weber forces are identical.

1. If the charge distribution contained within V is such as to render the body which that charge distribution describes to be electrically neutral, then it will be the case that the field term, $\boldsymbol{L}(\boldsymbol{x}, t, \boldsymbol{u})$,will be identically zero.
2. If it is also the case that the charge distribution contained within V also describes a closed conducting loop such that the thickness of the loop is small compared to its length, then it is straightforward to show that the field term $\boldsymbol{C}(\boldsymbol{x}, t, \boldsymbol{u})$ will also be identically zero since the integrand in $\boldsymbol{C}(\boldsymbol{x}, t, \boldsymbol{u})$ can be recast as a complete differential. See Appendix C.
3. Should it also be the case that the charge distribution contained within V' also renders the body which that distribution describes to be also electrically neutral, then it will also be the case that the external force component $\int_{\mathrm{V}'} \rho_e(\boldsymbol{x}, t)\boldsymbol{E}'(\boldsymbol{x}, t)\delta x_1 \delta x_2 \delta x_3$, due to the modified electric field, $\boldsymbol{E}'(\boldsymbol{x}, t)$,will be identically zero.

Under the conditions outlined above the total external force reduces to;

$$\boldsymbol{F}_{\mathrm{V}'} = \int_{\mathrm{V}'} \rho_e(\boldsymbol{x}, t)\boldsymbol{W}(\boldsymbol{x}, t, \boldsymbol{u})\delta x_1 \delta x_2 \delta x_3 = \boldsymbol{F}_{Weber} \qquad (1.27)$$

Therefore in conclusion, measuring experimentally such a force in which the above conditions are present makes it impossible to distinguish between the Weber and Lorentz force principles.

Evaluation of $\boldsymbol{E}(\boldsymbol{x}, t) = -\nabla\Phi(\boldsymbol{x}, \mathrm{t}) - \frac{\partial \mathbf{A}(\boldsymbol{x},t)}{\partial t}$. In a similar fashion the evaluation of $\boldsymbol{E}(\boldsymbol{x}, t)$ proceeds as follows. Utilizing Eq (A.1) from appendix A allows us to evaluate the term $\frac{\partial \boldsymbol{A}(\boldsymbol{x},t)}{\partial t}$ as;

$$\begin{aligned}\frac{\partial \boldsymbol{A}(\boldsymbol{x}, t)}{\partial t} &= \frac{1}{4\pi c^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y}, t)\frac{d}{dt}\Big(\frac{d\hat{\boldsymbol{r}}}{dt} + \hat{\boldsymbol{r}}\frac{\frac{dr}{dt}}{r}\Big)\delta y_1 \delta y_2 \delta y_3 \\ &= \frac{1}{4\pi c^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y}, t)\Big(\frac{d^2\hat{\boldsymbol{r}}}{dt^2} + \frac{d\hat{\boldsymbol{r}}}{dt}\frac{\frac{dr}{dt}}{r} + \hat{\boldsymbol{r}}\Big(\frac{1}{r}\frac{d^2 r}{dt^2} - \frac{1}{r^2}\Big(\frac{dr}{dt}\Big)^2\Big)\Big)\delta y_1 \delta y_2 \delta y_3 \qquad (1.28)\end{aligned}$$

Also using Eq (B.3) , Eq (B.6) and Eq (B.7) yields;

$$\begin{aligned}\nabla_{\boldsymbol{x}} \cdot \boldsymbol{A}(\boldsymbol{x}, t) &= \frac{1}{4\pi c^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y}, t)\nabla_{\boldsymbol{x}} \cdot \Big(\frac{d\hat{\boldsymbol{r}}}{dt} + \hat{\boldsymbol{r}}\frac{\frac{dr}{dt}}{r}\Big)\delta y_1 \delta y_2 \delta y_3 \\ &= \frac{1}{4\pi c^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y}, t)\frac{1}{r^2}\frac{dr}{dt}\delta y_1 \delta y_2 \delta y_3. \qquad (1.29)\end{aligned}$$

By employing Eq (A.1) and Eq (B.10) this then yields;

$$-\frac{1}{c^2}\frac{\partial^2\Phi(\boldsymbol{x},t)}{\partial t^2} = \frac{\partial(\nabla_{\boldsymbol{x}}\cdot\boldsymbol{A}(\boldsymbol{x},t))}{\partial t} = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\frac{d}{dt}(\frac{1}{r^2}\frac{dr}{dt})\delta y_1\delta y_2\delta y_3$$
$$= \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{1}{r^2}\frac{d^2r}{dt^2} - \frac{2}{r^3}\left(\frac{dr}{dt}\right)^2)\delta y_1\delta y_2\delta y_3$$
$$= -\frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\frac{1}{2}\nabla_{\boldsymbol{x}}{}^2(\frac{d^2r}{dt^2})\delta y_1\delta y_2\delta y_3$$
$$= -\frac{\nabla_{\boldsymbol{x}}{}^2}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\frac{1}{2}(\frac{d^2r}{dt^2})\delta y_1\delta y_2\delta y_3. \qquad (1.30)$$

Consequently and also because $\Phi(\boldsymbol{x},t)$ obeys the wave equation it must be the case that;

$$\nabla_{\boldsymbol{x}}{}^2\Phi(\boldsymbol{x},t) - \frac{1}{c^2}\frac{\partial^2\Phi(\boldsymbol{x},t)}{\partial t^2} = \nabla_{\boldsymbol{x}}{}^2\left(\Phi(\boldsymbol{x},t) - \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\frac{1}{2}\left(\frac{d^2r}{dt^2}\right)\delta y_1\delta y_2\delta y_3\right)$$
$$= -\rho_e(\boldsymbol{x},t) \qquad (1.31)$$

which in turn implies that;

$$\Phi(\boldsymbol{x},t) - \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\frac{1}{2}\left(\frac{d^2r}{dt^2}\right)\delta y_1\delta y_2\delta y_3 = \frac{1}{4\pi}\int_V \rho_e(\boldsymbol{y},t)\frac{1}{r}\delta y_1\delta y_2\delta y_3, \quad (1.32)$$

i.e.

$$\Phi(\boldsymbol{x},t) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{c^2}{r} + \frac{1}{2}\left(\frac{d^2r}{dt^2}\right))\delta y_1\delta y_2\delta y_3. \qquad (1.33)$$

Eq (1.33) takes into account the radiation term $\frac{1}{c^2}\frac{\partial^2\Phi(\boldsymbol{x},t)}{\partial t^2}$ which is clearly of order $O\left(\frac{1}{c^2}\right)$, and as such cannot be ignored. Using Eq (1.33) then allows the gradient term $\nabla_{\boldsymbol{x}}\Phi(\boldsymbol{x},t)$, to be calculated as;

$$\nabla_{\boldsymbol{x}}\Phi(\boldsymbol{x},t) = \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\nabla_{\boldsymbol{x}}(\frac{c^2}{r} + \frac{1}{2}\left(\frac{d^2r}{dt^2}\right))\delta y_1\delta y_2\delta y_3$$
$$= \frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{c^2}{r^2}\hat{\boldsymbol{r}} - \frac{1}{2}\left(\frac{d^2\hat{\boldsymbol{r}}}{dt^2}\right))\delta y_1\delta y_2\delta y_3. \qquad (1.34)$$

Finally combining Eq (1.28) and Eq (1.34) then allows the final form of $\boldsymbol{E}(\boldsymbol{x},\mathrm{t})$ to be written as;

$$\boldsymbol{E}(\boldsymbol{x},\mathrm{t}) = -\frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{d^2\hat{\boldsymbol{r}}}{dt^2} + \frac{d\hat{\boldsymbol{r}}}{dt}\frac{\frac{dr}{dt}}{r} + \hat{\boldsymbol{r}}(\frac{1}{r}\frac{d^2r}{dt^2} - \frac{1}{r^2}(\frac{dr}{dt})^2))\delta y_1\delta y_2\delta y_3$$
$$-\frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)(\frac{c^2}{r^2}\hat{\boldsymbol{r}} - \frac{1}{2}\left(\frac{d^2\hat{\boldsymbol{r}}}{dt^2}\right))\delta y_1\delta y_2\delta y_3$$
$$= -\frac{1}{4\pi c^2}\int_V \rho_e(\boldsymbol{y},t)\left(\frac{1}{2}\frac{d^2\hat{\boldsymbol{r}}}{dt^2} + \frac{d\hat{\boldsymbol{r}}}{dt}\frac{\frac{dr}{dt}}{r}\right.$$
$$\left. + \hat{\boldsymbol{r}}\left(\frac{c^2}{r^2} - \frac{1}{r^2}\left(\frac{dr}{dt}\right)^2 + \frac{1}{r}\frac{d^2r}{dt^2}\right)\right)\delta y_1\delta y_2\delta y_3 \qquad (1.35)$$

Consequently from Eq (1.24) we can calculate $\boldsymbol{E}'(\boldsymbol{x},\mathrm{t})$ as;

$$\boldsymbol{E}'(\boldsymbol{x},\mathrm{t}) = \frac{1}{4\pi\mathrm{c}^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)(\frac{\hat{\boldsymbol{r}}}{r^2}\left(\mathrm{c}^2 + r\frac{d^2r}{dt^2} - \frac{1}{2}\left(\frac{dr}{dt}\right)^2\right)\delta y_1\delta y_2\delta y_3$$
$$- \frac{1}{4\pi\mathrm{c}^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)\left(\frac{1}{2}\frac{d^2\hat{\boldsymbol{r}}}{dt^2} + \frac{d\hat{\boldsymbol{r}}}{dt}\frac{\frac{dr}{dt}}{r} + \hat{\boldsymbol{r}}\left(\frac{\mathrm{c}^2}{r^2} - \frac{1}{r^2}\left(\frac{dr}{dt}\right)^2 + \frac{1}{r}\frac{d^2r}{dt^2}\right)\right)\delta y_1\delta y_2\delta y_3$$

$$= -\frac{1}{4\pi\mathrm{c}^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)\left(\frac{1}{2}\frac{d^2\hat{\boldsymbol{r}}}{dt^2} + \frac{d\hat{\boldsymbol{r}}}{dt}\frac{\frac{dr}{dt}}{r} - \frac{1}{2}\frac{1}{r^2}\left(\frac{dr}{dt}\right)^2\hat{\boldsymbol{r}}\right)\delta y_1\delta y_2\delta y_3$$
$$= \frac{1}{4\pi\mathrm{c}^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)\nabla_{\boldsymbol{x}}\left(\frac{1}{2}\frac{d^2r}{dt^2} + \frac{1}{2}\frac{1}{r}\left(\frac{dr}{dt}\right)^2\right)\delta y_1\delta y_2\delta y_3 = -\nabla_{\boldsymbol{x}}\Phi'(\boldsymbol{x},t) \quad (1.36)$$

in which

$$\Phi'(\boldsymbol{x},t) = -\frac{1}{4\pi\mathrm{c}^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)\left(\frac{1}{2}\frac{d^2r}{dt^2} + \frac{1}{2}\frac{1}{r}\left(\frac{dr}{dt}\right)^2\right)\delta y_1\delta y_2\delta y_3 \qquad (1.37)$$

and is termed the Modified Electric Scalar Potential.

Induced EMF In a Closed Loop Conductor. Here we will calculate the EMF induced in a closed loop conductor due to a source distribution which satisfies conditions 1 and 2. Since it is the case that for the vector line element, $\delta\boldsymbol{l}'$, on the curve, C', which describes the geometry of a closed loop conductor, the term, $\left(\boldsymbol{u}(\boldsymbol{x}(l'),t)\times\boldsymbol{B}(\boldsymbol{x}(l'),t)\right).\delta\boldsymbol{l}'$, is zero ,then the induced EMF in such a conductor can be written as;

$$EMF = \int_{C'} \boldsymbol{E}(\boldsymbol{x},\mathrm{t}).\delta\boldsymbol{l}' = \int_{C'} (\boldsymbol{E}(\boldsymbol{x}(l'),t) + \boldsymbol{u}(\boldsymbol{x}(l'),t)\times\boldsymbol{B}(\boldsymbol{x}(l'),t)).\delta\boldsymbol{l}'$$
$$= \int_{C'} (\boldsymbol{W}(\boldsymbol{x}(l'),t,\boldsymbol{u}(\boldsymbol{x}(l'),t)) + \boldsymbol{E}'(\boldsymbol{x}(l'),t)).\delta\boldsymbol{l}'$$
$$= \int_{C'} (\boldsymbol{W}(\boldsymbol{x}(l'),t,\boldsymbol{u}(\boldsymbol{x}(l'),t)) - \nabla_{\boldsymbol{x}}\Phi'(\boldsymbol{x}(l'),t)).\delta\boldsymbol{l}'$$
$$= \int_{C'} (\boldsymbol{W}(\boldsymbol{x}(l'),t,\boldsymbol{u}(\boldsymbol{x}(l'),t)).\delta\boldsymbol{l}' \qquad (1.38).$$

Specifically as $\boldsymbol{u}(\boldsymbol{x}(l'),t)$ represents the velocity field of the charge carriers in the conductor then the vectors $\boldsymbol{u}(\boldsymbol{x},\mathrm{t})$ and $\delta\boldsymbol{l}'$ are hence collinear, thus rendering the term,

$$\left(\boldsymbol{u}(\boldsymbol{x}(l'),t)\times\boldsymbol{B}(\boldsymbol{x}(l'),t)\right).\delta\boldsymbol{l}'$$

equal to zero. Again we see, as in the case of the mechanical force operating on a current element, the solitary presence of the Weber field, $\boldsymbol{W}$.

Direct comparison between Lorentz's Force and Weber's Force. If one were to assume that the Weber Force was generally true under all conditions then the framework developed in the preceding analysis allows for a very insightful comparison with the Lorentz force, when both force principles are expressed in terms of the same vector and distance quantities, $\{\frac{d^2\hat{\boldsymbol{r}}}{dt^2}\ \frac{d\hat{\boldsymbol{r}}}{dt}\ \hat{\boldsymbol{r}}\}$ and $\{r, \frac{dr}{dt}, \frac{d^2r}{dt^2}\}$ respectively. After combing Eq (1.35) and Eq (1.13), for the Lorentz Force we have;

$$\boldsymbol{F}_{V'} = \boldsymbol{F}_{Lorentz} = -\frac{1}{4\pi c^2}\int_{V'} \rho_e(\boldsymbol{x},t)(\int_{V} \rho_e(\boldsymbol{y},t)\left(\frac{1}{2}\frac{d^2\hat{\boldsymbol{r}}}{dt^2} + \frac{d\hat{\boldsymbol{r}}}{dt}\frac{1}{r}\left(\frac{dr}{dt} + (\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\right)\right.$$
$$\left. + \frac{\hat{\boldsymbol{r}}}{r^2}\left(c^2 - \left(\frac{dr}{dt}\right)^2 + r\frac{d^2r}{dt^2} - \boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot r\frac{d\hat{\boldsymbol{r}}}{dt}\right)\right)\delta y_1\delta y_2\delta y_3)\delta x_1\delta x_2\delta x_3 \qquad (1.39)$$

Similarly, and this time by employing Eq (1.27) and expanding the terms out in terms of the above mentioned vector and distance quantities, we have for the Weber Force;

$$\boldsymbol{F}_{V'} = \boldsymbol{F}_{Weber} = -\frac{1}{4\pi c^2}\int_{V'} \rho_e(\boldsymbol{x},t)(\int_{V} \rho_e(\boldsymbol{y},t)\left(\frac{\hat{\boldsymbol{r}}}{r^2}\left(c^2 - \frac{1}{2}(\frac{Dr}{Dt})^2\right.\right.$$
$$\left.\left. + r\frac{D^2r}{Dt^2}\right)\right)\delta y_1\delta y_2\delta y_3)\delta x_1\delta x_2\delta x_3$$
$$= -\frac{1}{4\pi c^2}\int_{V'} \rho_e(\boldsymbol{x},t)(\int_{V} \rho_e(\boldsymbol{y},t)\left(\frac{\hat{\boldsymbol{r}}}{r^2}\left(c^2 - \frac{1}{2}\left(\frac{dr}{dt}\right)^2 + r\frac{d^2r}{dt^2} - \boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot r\frac{d\hat{\boldsymbol{r}}}{dt}\right.\right.$$
$$+ \frac{dr}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}}) - \frac{1}{2}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})^2 - r\frac{d}{dt}(\boldsymbol{u}(\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}}) - \boldsymbol{u}(\boldsymbol{x},\mathrm{t})$$
$$\left.\left.\cdot r\nabla_x(\boldsymbol{u}\cdot\hat{\boldsymbol{r}})\right)\right)\delta y_1\delta y_2\delta y_3)\delta x_1\delta x_2\delta x_3 \qquad (1.40)$$

Experimentally it should in principle be possible to distinguish one force from the other by violating within an experimental setup the conditions described in the previous section which rendered the two force laws to be identical. Perhaps the most efficient means of violating the equality conditions as described above, should occur in a plasma, with the most observable qualitative features of plasma behaviour offering a distinct signature of support for the actual physical principle operating in nature, and hence to be able to establish as to whether that principle leans more in support of the Weber or the Lorentz-Maxwell paradigm.

Conclusion

Probably the most contentious objection to Weber's theory of Electrodynamics from the proponents of Maxwell's theory has always been the assertion that the Weber theory doesn't provide a mechanism for radiation which is partly due to the conceptual nature of Weber's theory being based on direct particle to particle interaction. However as has been established by Eq (1.33), at least to order $O(\frac{1}{c^2})$, the inclusion of radiation effects in the Maxwell-Lorentz theory still leads to the same predictions as the Weber theory in so far as their respective descriptions of induction phenomena and the mechanical forces exerted on current carrying elements. As has been emphasised, in order to experimentally discriminate between each theory, it will be necessary to violate any of conditions 1, 2 and 3. As has already been suggested, one approach could be based on using each theory to model plasma behaviour which is then compared with actual observations.

In regard to applications to the modelling of circuit behaviour, it is felt on the basis of the results established by this investigation, that Weber's theory should be adequate. This has important implications for the Integrated Circuit industry as the conventional Spice approach is no longer adequate at the component scales characteristic of today's IC's. These deficiencies are resolved by the coupling of the Spice approach with full blown simulations of Maxwell's Equations in order to satisfactorily describe chip behaviour to industry standards. However the drawback with this approach is that the 3-D mesh densities required for the finite difference schemes are so high, that many CPU hours of computational resources are required for each simulation which then adds to the overall cost of the product. Weber's theory could potentially address this issue as it is essentially, at least in regard to this application, a one dimensional problem, since all spatial integrations can be approximated as one dimensional computations. The reason for this is that the bulk of the correction required to augment the Spice predictions are due to the interference effects between component connectors, which are to all intents and purposes, one dimensional entities.

The methodology adopted in this investigation can be applied to other force laws as a means of comparison to the Weber theory and the Maxwell-Lorentz theory, and will in fact form the body of a future research effort at Forbin Systems.

Appendix A

if it's assumed that $\frac{\partial \rho_e(y,t)}{\partial t} + \nabla_{\mathrm{y}}.\left(\rho_e(\boldsymbol{y},t)u(\boldsymbol{y},t)\right) = 0$ then from Eq (1.6) we have the result that;

$$
\begin{aligned}
\frac{\partial}{\partial t}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t) f(\boldsymbol{x},\boldsymbol{y},t)\delta y_1 \delta y_2 \delta y_3 &= \int_{\mathrm{V}} \frac{\partial(\rho_e(\boldsymbol{y},t) f(\boldsymbol{x},\boldsymbol{y},t))}{\partial t}\delta y_1 \delta y_2 \delta y_3 \\
&= \int_{\mathrm{V}} (\rho_e \frac{\partial f}{\partial t} + f\frac{\partial \rho_e}{\partial t})\delta y_1 \delta y_2 \delta y_3 = \int_{\mathrm{V}} (\rho_e \frac{\partial f}{\partial t} - f\nabla_{\mathrm{y}}.(\rho_e u(\boldsymbol{y},t)))\delta y_1 \delta y_2 \delta y_3 \\
&= \int_{\mathrm{V}} (\rho_e \frac{\partial f}{\partial t} - f\nabla_{\mathrm{y}}.(\rho_e u(\boldsymbol{y},t)))\delta y_1 \delta y_2 \delta y_3 \\
&= \int_{\mathrm{V}} (\rho_e \frac{\partial f}{\partial t} - \nabla_{\mathrm{y}}.\left(\mathrm{f}\,\rho_e u(\boldsymbol{y},t)\right) + \rho_e u(\boldsymbol{y},t).\nabla_{\mathrm{y}}\mathrm{f})\delta y_1 \delta y_2 \delta y_3 \\
&= \int_{\mathrm{V}} \rho_e \frac{df}{dt}\delta y_1 \delta y_2 \delta y_3 - \oiiint_{\Omega} (\mathrm{f}\,\rho_e u(\boldsymbol{y},t)).dS = \int_{\mathrm{V}} \rho_e \frac{df}{dt}\delta y_1 \delta y_2 \delta y_3 - 0 \\
&= \int_{\mathrm{V}} \rho_e \frac{df}{dt}\delta y_1 \delta y_2 \delta y_3 \qquad (A.1)
\end{aligned}
$$

Provided that the containing volume V to be of sufficient extent such that on the surface, Ω , enclosing V , we have the condition that $\rho_e(\boldsymbol{y},t) = 0\ for\ all\ \boldsymbol{y} \in \Omega$.

Appendix B

$$\nabla_{\boldsymbol{x}} \times \hat{\boldsymbol{r}} = 0 \qquad (B.1)$$

$$\nabla_{\boldsymbol{x}} \times \frac{d\hat{\boldsymbol{r}}}{dt} = \frac{d(\nabla_{\boldsymbol{x}} \times \hat{\boldsymbol{r}})}{dt} = 0 \quad (B.2)$$

$$\nabla_{\boldsymbol{x}}\, r = -\hat{\boldsymbol{r}} \qquad (B.3)$$

$$\nabla_{\boldsymbol{x}} \frac{dr}{dt} = \frac{d(\nabla_{\boldsymbol{x}}\, r)}{dt} = -\frac{d\hat{\boldsymbol{r}}}{dt} \qquad (B.4)$$

$$\frac{d\hat{\boldsymbol{r}}}{dt} \cdot \hat{\boldsymbol{r}} = \frac{1}{2}\frac{d(\hat{\boldsymbol{r}}.\hat{\boldsymbol{r}})}{dt} = \frac{1}{2}\frac{d1}{dt} = 0 \qquad (B.5)$$

$$\nabla_{\boldsymbol{x}} \cdot \hat{\boldsymbol{r}} = -\frac{2}{r} \qquad (B.6)$$

$$\nabla_{\boldsymbol{x}} \cdot \frac{d\hat{\boldsymbol{r}}}{dt} = \frac{d(\nabla_{\boldsymbol{x}} \cdot \hat{\boldsymbol{r}})}{dt} = \frac{2}{r^2}\frac{dr}{dt} \qquad (B.7)$$

$$\nabla_{\boldsymbol{x}} \cdot \frac{d^2\hat{\boldsymbol{r}}}{dt^2} = \frac{d^2(\nabla_{\boldsymbol{x}} \cdot \hat{\boldsymbol{r}})}{dt^2} = \frac{d}{dt}\left(\frac{2}{r^2}\frac{dr}{dt}\right) = 2\left(\frac{1}{r^2}\frac{d^2r}{dt^2} - \frac{2}{r^3}\left(\frac{dr}{dt}\right)^2\right) = 2\frac{d}{dt}\left(\frac{1}{r^2}\frac{dr}{dt}\right) \qquad (B.8)$$

$$\nabla_{\boldsymbol{x}} \cdot \left(\frac{1}{r}\frac{dr}{dt}\hat{\boldsymbol{r}}\right) = \frac{1}{r}\frac{dr}{dt}\nabla_{\boldsymbol{x}} \cdot \hat{\boldsymbol{r}} + \hat{\boldsymbol{r}}.\nabla_{\boldsymbol{x}}\left(\frac{1}{r}\frac{dr}{dt}\right) = \frac{1}{r}\frac{dr}{dt}\nabla_{\boldsymbol{x}} \cdot \hat{\boldsymbol{r}} + \hat{\boldsymbol{r}}.\left(-\frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt} + \frac{1}{r^2}\frac{dr}{dt}\hat{\boldsymbol{r}}\right) = -\frac{2}{r^2}\frac{dr}{dt} + \frac{1}{r^2}\frac{dr}{dt}$$
$$= -\frac{1}{r^2}\frac{dr}{dt} \qquad (B.9)$$

$$\nabla_{\boldsymbol{x}}{}^2\left(\frac{d^2r}{dt^2}\right) = \nabla_{\boldsymbol{x}} \cdot \left(\nabla_{\boldsymbol{x}}\frac{d^2r}{dt^2}\right) = \nabla_{\boldsymbol{x}} \cdot \left(\frac{d^2(\nabla_{\boldsymbol{x}}\, r)}{dt^2}\right) = -\nabla_{\boldsymbol{x}} \cdot \frac{d^2\hat{\boldsymbol{r}}}{dt^2}$$
$$= -2\left(\frac{1}{r^2}\frac{d^2r}{dt^2} - \frac{2}{r^3}\left(\frac{dr}{dt}\right)^2\right) \qquad (B.10)$$

$$\nabla_{\boldsymbol{x}} \times \left(\frac{d\hat{\boldsymbol{r}}}{dt} + \hat{\boldsymbol{r}}\frac{\frac{dr}{dt}}{r}\right) = \nabla_{\boldsymbol{x}} \times \left(\hat{\boldsymbol{r}}\frac{\frac{dr}{dt}}{r}\right) = \frac{\frac{dr}{dt}}{r}(\nabla_{\boldsymbol{x}} \times \hat{\boldsymbol{r}}) + \nabla_{\boldsymbol{x}}\left(\frac{\frac{dr}{dt}}{r}\right) \times \hat{\boldsymbol{r}} = \nabla_{\boldsymbol{x}}\left(\frac{\frac{dr}{dt}}{r}\right) \times \hat{\boldsymbol{r}}$$
$$= \left(\frac{dr}{dt}\nabla_{\boldsymbol{x}}\left(\frac{1}{r}\right) + \frac{1}{r}\nabla_{\boldsymbol{x}}\left(\frac{dr}{dt}\right)\right) \times \hat{\boldsymbol{r}} = \left(\frac{1}{r^2}\frac{dr}{dt}\hat{\boldsymbol{r}} - \frac{1}{r}\frac{\partial\hat{\boldsymbol{r}}}{\partial t}\right) \times \hat{\boldsymbol{r}} = \hat{\boldsymbol{r}} \times \frac{1}{r}\frac{d\hat{\boldsymbol{r}}}{dt} \qquad (B.11)$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c} \qquad (B.12)$$

$$\frac{1}{2}\nabla_{\boldsymbol{x}}\left(\frac{1}{r}\left(\frac{dr}{dt}\right)^2\right) = -\left(\frac{1}{r}\frac{dr}{dt}\frac{d\hat{\boldsymbol{r}}}{dt} - \frac{1}{2}\frac{1}{r^2}\left(\frac{dr}{dt}\right)^2\right)\hat{\boldsymbol{r}} \quad (B.13)$$

$$-\nabla_{\boldsymbol{x}}\left(\frac{d^2r}{dt^2}\right) = \frac{d^2\hat{\boldsymbol{r}}}{dt^2} \qquad (B.14)$$

$$\nabla_{\boldsymbol{x}}\left(\frac{1}{r}\right) = \frac{1}{r^2}\hat{\boldsymbol{r}} \qquad (B.15)$$

Appendix C

If it is also the case that the charge distribution contained within V also describes a closed conducting loop such that the thickness of the loop is small compared to its length, then it is straightforward to show that the field term $\boldsymbol{C}(\boldsymbol{x},t,\boldsymbol{u})$ will also be identically zero since the integrand in $\boldsymbol{C}(\boldsymbol{x},t,\boldsymbol{u})$ can be recast as a complete differential. This is easily demonstrated in the following.

$$\boldsymbol{C}(\boldsymbol{x},t,\boldsymbol{u}) = \frac{1}{4\pi c^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)\nabla_{\boldsymbol{y}}\left((\boldsymbol{u}(\,\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)\cdot\boldsymbol{u}(\,\boldsymbol{y},\mathrm{t})\delta y_1\delta y_2\delta y_3$$

$$= \frac{1}{4\pi c^2}\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)\frac{\Delta\left((\boldsymbol{u}(\,\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)}{\Delta t}\delta y_1\delta y_2\delta y_3 \qquad (C.1)$$

$$\Delta\left((\boldsymbol{u}(\,\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right) = \nabla_{\boldsymbol{y}}\left((\boldsymbol{u}(\,\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)\cdot\Delta\boldsymbol{y} \quad (C.2)$$

Now since the conductor can be approximately described as a one dimensional entity, then the position vector $\boldsymbol{y}$,referring to a point within V ,can be parameterized as $\boldsymbol{y}(l)$ in which l is a measure of the distance along the conductor in a clockwise direction from some arbitrary point on the curve describing the shape of the conducting loop . Also $A(l)$ will refer to the cross sectional area of the conductor at the point as indicated by $\boldsymbol{y}(l)$. With these definitions and conventions in mind the volume element, $\delta y_1\delta y_2\delta y_3$, therefore becomes simply $A(l)\delta l$. To wit we have;

$$\int_{\mathrm{V}} \rho_e(\boldsymbol{y},t)\frac{\Delta\left((\boldsymbol{u}(\,\boldsymbol{x},\mathrm{t})\cdot\hat{\boldsymbol{r}})\frac{\hat{\boldsymbol{r}}}{r}\right)}{\Delta t}\delta y_1\delta y_2\delta y_3$$

$$= \int_0^L \rho_e(\boldsymbol{y}(l),t)\frac{\Delta\left(\left(\boldsymbol{u}(\,\boldsymbol{y}(l),\mathrm{t})\cdot\hat{\boldsymbol{r}}\big(\boldsymbol{x},\boldsymbol{y}(l)\big)\right)\frac{\hat{\boldsymbol{r}}\big(\boldsymbol{x},\boldsymbol{y}(l)\big)}{r\big(\boldsymbol{x},\boldsymbol{y}(l)\big)}\right)}{\Delta l}\frac{\Delta l}{\Delta t}A(l)\delta l \quad with \;\; 0\le l \le L \qquad (C.3)$$

$$\rho_e(\boldsymbol{y}(l),t)\frac{\Delta l}{\Delta t}A(l) = I(t) \quad (C.4)$$

in which L indicats the total length of the conductor loop, and $I(t)$ refers the charge current at time t. Consequently we have the result;

$$\int_0^L \rho_e(\boldsymbol{y}(l),t)\frac{\Delta\left((\boldsymbol{u}(\,\boldsymbol{y}(l),\mathrm{t})\cdot\hat{\boldsymbol{r}}(\boldsymbol{x},\boldsymbol{y}(l)))\frac{\hat{\boldsymbol{r}}(\boldsymbol{x},\boldsymbol{y}(l))}{r(\boldsymbol{x},\boldsymbol{y}(l))}\right)}{\Delta l}\frac{\Delta l}{\Delta t}A(l)\delta l$$
$$= \int_0^L I(t)\frac{\Delta\left((\boldsymbol{u}(\,\boldsymbol{y}(l),\mathrm{t})\cdot\hat{\boldsymbol{r}}(\boldsymbol{x},\boldsymbol{y}(l)))\frac{\hat{\boldsymbol{r}}(\boldsymbol{x},\boldsymbol{y}(l))}{r(\boldsymbol{x},\boldsymbol{y}(l))}\right)}{\Delta l}\delta l$$
$$= I(t)\int_0^L \Delta\left((\boldsymbol{u}(\,\boldsymbol{y}(l),\mathrm{t})\cdot\hat{\boldsymbol{r}}(\boldsymbol{x},\boldsymbol{y}(l)))\frac{\hat{\boldsymbol{r}}(\boldsymbol{x},\boldsymbol{y}(l))}{r(\boldsymbol{x},\boldsymbol{y}(l))}\right) = 0 \quad with \;\; 0 \le l \le L \quad (C.5).$$

Allowing us to conclude that;

$$\boldsymbol{C}(\boldsymbol{x},t,\boldsymbol{u}) = 0 \quad (C.6)$$

QED.

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